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Data-driven Identification of Encoding on Quadratic-Manifolds for High-Fidelity Dynamical Models

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Most model order reduction (MOR) approaches for dynamical systems of the form:

$$\dot{x}(t) = f(x(t)), \quad x(0) \in \mathbb{R}^n$$

seek for an efficient encoding of the variables of a dynamical system on a linear manifold. Given, for example, a *Proper Orthogonal Decomposition* (POD) basis $V \in \mathbb{R}^{n,k}$, the variable $x(t) \in \mathbb{R}^n$ is encoded as $\hat{x}(t) \in \mathbb{R}^k$ and decoded as $\tilde{x}(t) \in \mathbb{R}^n$ via the relations

$$\hat{x}(t) = V^T x(t) \quad \text{and} \quad \tilde{x}(t) = V\hat{x}(t) \approx x(t).$$

While the POD is optimal in representing a given set of solution snapshots on a linear manifold, in many applications such as transport dominant ones, the POD coordinates $\hat{x}(t)$ with a linear embedding V fail to encode complex dynamics accurately. To overcome the general limitation of these linear projection methods, one may think of parametrizing the dynamics x on a low-dimensional manifold $M \subset \mathbb{R}^k$ with a nonlinear mapping:

$$\rho\colon \mathbb{R}^k \to \mathbb{R}^n\colon \hat{x}(t) \to \tilde{x}(t).$$

Such a nonlinear parametrization can then be used for a reduced-order model via

$$\tfrac{d}{dt}(\rho(\hat{x}(t))) = f(\rho(\hat{x}(t))), \quad \hat{x}(0) \in \mathbb{R}^k.$$

As in [1], we consider a linear-quadratic model for the map ρ:

$$\rho\colon \hat{x}(t) \to \Omega(\hat{x}(t) \otimes \hat{x}(t)) + V\hat{x}(t),$$

and as in [2], we propose a data-driven approach for identifying Ω and V.

In this contribution, we explore setups that simultaneously determine $\hat{x}$ and the decoding through V and Ω. We illustrate the performance of the quadratic manifold to design reduced-order models for fluid flows and compare to standard linear methods.

REFERENCES

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