

Solving parametric PDEs with an enhanced model reduction method based on Linear/Ridge expansions

Constantin Greif^{*1}, Philipp Junk² and Karsten Urban¹

¹ Ulm University, Institute of Numerical Mathematics, Helmholtzstr. 20, 89081 Ulm (Germany), {constantin.greif, karsten.urban}@uni-ulm.de

² Justus-Liebig-University Giessen, Lehrstuhl Numerische Mathematik, Arndtstr. 2, 35392 Giessen (Germany), philipp.junk@math.uni-giessen.de

Keywords: *Nonlinear Model Reduction, Parametrized Partial Differential equations, Hyperbolic equations, Particle swarm optimization, Ridge function approximation*

Classical projection-based model reduction methods, like the reduced basis method [1, 2], are popular tools for getting quickly solvable reduced order models for parametric PDEs. Unfortunately, for some problems the error-decay with respect to the dimension of the projection space is predetermined to be slow, e.g., for transport or wave equations with jump discontinuities [2]. Therefore nonlinear methods are needed that do not project onto a linear space.

Our approach is to build the reduced order model with linear basis functions that are enriched with some ridge functions [3, 4]. A ridge function consists of a one dimensional profile function as well as a direction that expands the profile to the multivariate domain. For searching the optimal directions, we developed a particle grid algorithm. The Linear/Ridge function space of linear basis functions as well as linear combinations of ridge functions with parameter-dependent directions, is clearly nonlinear.

Offline, we first build the linear basis with known techniques by using snapshots that are solutions of the parametric PDE. If the error does not decay fast enough by adding more basis functions, we enrich the linear space with ridge functions. This is done by iteratively picking a worst parameter, computing the associated snapshot and determining directions for the profiles to approximate the snapshot at best. Then, extending the ridge function space by adding new profiles to the basis set until the approximation is precise enough. Finally with the generated Linear/Ridge function space, the parametric PDE can be solved online-efficient by using interpolation for the directions.

REFERENCES

- [1] Jan S. Hesthaven, Gianluigi Rozza and Benjamin Stamm. Certified Reduced Basis Methods for Parametrized Partial Differential Equations. *Springer*, 2016.
- [2] Mario Ohlberger and Stephan Rave. Reduced Basis Methods: Success, Limitations and Future Challenges. *Proceedings of the Conference Algoritmy*, pp. 1–12, 2016.
- [3] Constantin Greif, Philipp Junk and Karsten Urban. Linear/Ridge Expansions: Enhancing Linear Approximations By Ridge Functions. *arXiv, submitted to Adv. Comp. Math*, 2021.
- [4] Alan Pinkus. Ridge Functions. *Cambridge University Press*, 2015.