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Stochastic modelling of thermal effects on a ferromagnetic nano particle

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In this work, we study the behaviour of a single ferromagnetic nano particle submitted to an external field and a stochastic perturbation. Consider a magnetic moment $\mu : \mathbb{R} \to S_2$ submitted to an external field $b \in \mathbb{R}^3$, where S_2 denotes the unit sphere $\mathbb{R}^3$. The evolution of μ is controlled by the Landau-Lifchitz equation

$$\frac{d\mu}{dt} = -\mu \wedge b - \alpha\mu \wedge (\mu \wedge b), \quad \mu(0) \in S_2$$

where $\alpha > 0$ is the damping coefficient. The stochastic model must satisfy the fondamental property of the magnetic moment that its norm is constant over time, $|\mu(t)| = 1$ for all $t \geq 0$. Then, we consider the following stochastic model

$$d\mu_t = -\mu_t \wedge (b\ dt + \varepsilon\ dW_t) - \alpha\mu_t \wedge (\mu_t \wedge (b\ dt + \varepsilon_t\ dW_t)) + K_t dt \tag{1}$$

where W is a Brownian motion in $\mathbb{R}^3$ and ε_t is a non-negative deterministic and decreasing function. The adapted process K is a pull back term to ensure that μ_t stays on the unit sphere and that therefore its norm is contant over time. We prove that the process K must be of the form

$$K_t = -\varepsilon_t^2(\alpha^2 + 1)\mu_t + \mu_t^\perp$$

where $\mu_t^\perp$ is an adapted process orthogonal to μ_t for all t.

In [1], we studied a slightly different approach obtained by rescaling μ instead of introducing a pullback term K. In this talk, we will make the connection between the two approaches. We will also discuss the long time stabilization results obtained in [2] stating the almost sure convergence of μ to $b/|b|$ if ε decreases fast enough and controlling the rate of convergence in the L^p norm.

REFERENCES

[1] P. Etoré, S. Labbé, and J. Lelong. *Long time behaviour of a stochastic nano particle. Journal of Differential Equations*, 257(6):2115–2135, 2014.

[2] S. Labbé, and J. Lelong. *Stochastic modelling of thermal effects on a ferromagnetic nano particle. Journal of Dynamics and Differential Equations*, 32, 2020.