

A Stable Mixed Finite Element Method for the Elastic Deformation of Coastal Structures

Eirik Valseth^{1*}, Clint Dawson², Albert Romkes³ and Austin Kaul⁴

¹ The Oden Institute at the University of Texas at Austin, 201 East 24th street Stop C0200, Austin, TX 78712, USA, eirik@oden.utexas.edu

² The Oden Institute at the University of Texas at Austin, 201 East 24th street Stop C0200, Austin, TX 78712, USA, clint@oden.utexas.edu

³ South Dakota School of Mines and Technology, 501 East St. Joseph street, Rapid City, SD 57701, USA, albert.romkes@sdsmt.edu

⁴ South Dakota School of Mines and Technology, 501 East St. Joseph street, Rapid City, SD 57701, USA, austin.kaul@mines.sdsmt.edu

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The use of materials such as polymers and composites in coastal structures and engineering applications is highly desirable due to their many advantages compared to classical engineering materials. However, in cases where the polymer or composite constituents are nearly incompressible, stability issues arise in classical finite element (FE) methods. Additionally, the irregular loads encountered in many coastal structures can also lead to numerically unstable FE discretizations. To overcome these stability issues, we introduce a mixed FE method which result in numerically stable discretizations. Furthermore, the weak formulation of this method is posed such that the displacement is in H^1 and the stress tensor is in $H(\text{div})$, i.e., both are continuous. We present multiple numerical verifications, including comparison to the Galerkin FE method as well as other stable mixed methods. The method also includes a built-in *a posteriori* error estimate which is exploited in adaptive mesh refinement strategies. Finally, we present a verification of adaptive mesh refinements for a composite material that consist of a nearly incompressible polymer matrix to accurately resolve the stress field near an inclusion of a stiff material.