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Rare event probability estimation through high-dimensional elliptical distribution modeling

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Computing the probability that an engineering system reaches a particular failure state is a major challenge in reliability analysis. It requires the evaluation of a multiple integral over a complex set describing detrimental configurations of the system random inputs. The present work focuses on systems whose inputs can be modeled using elliptical laws.

Importance sampling (IS) is a simulation technique based on the Monte Carlo method in which an auxiliary density is introduced in the computation of the probability integral to reduce the variance of the probability estimator. To this end, the cross entropy (CE) method constructs a near optimal auxiliary density with an adaptive sampling approach by minimizing the Kullback–Leibler divergence between the theoretically optimal auxiliary density and a suitably chosen parametric family of distributions. However, most parametric densities in high dimension (several hundred inputs) are not flexible enough to account for the shape of the failure domain, causing the IS weights to collapse.

As an alternative, it is proposed here to construct a density for each failure zone in high-dimensional space, assuming the gradient of the system to be known. The global auxiliary IS density is then taken to be a mixture of all those densities. Using the stochastic decomposition of elliptical distributions, each failure zone density is modeled as the product of a conditional density for the radial component over the importance ring and a von Mises–Fisher density for the directional component, optimized with the CE method [2]. This approach is shown to enhance flexibility and accuracy in high-dimensional IS as the radius and direction of the elliptical random vector of system inputs can be optimized jointly for each failure zone while remaining stochastically independent [1].

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