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Spectral coarse spaces for indefinite and non-self adjoint problems

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GenEO ('Generalised Eigenvalue problems on the Overlap') [1] is a method for computing an operator-dependent spectral coarse space to be combined with local solves on subdomains to form a robust parallel domain decomposition preconditioner for elliptic PDEs. It has previously been proved, in the self-adjoint and positive-definite case, that this method, when used as a preconditioner for conjugate gradients, yields iteration numbers which are completely independent of the heterogeneity of the coefficient field of the partial differential operator. We extend this theory to the case of convection–diffusion–reaction problems (class in which we also include low-frequency Helmholtz problems), which may be non-self-adjoint and indefinite, and whose discretisations are solved with preconditioned GMRES. The GenEO coarse space is defined here using a generalised eigenvalue problem based on a self-adjoint and positive-definite subproblem. We obtain GMRES iteration counts which are independent of the variation of the coefficient of the diffusion term in the operator and depend only very mildly on the variation of the other coefficients. While the iteration number estimates do grow as the non-self-adjointness and indefiniteness of the operator increases, practical tests indicate the deterioration is much milder. Thus we obtain an iterative solver which is efficient in parallel and very effective for a wide range of convection–diffusion– reaction problems. At the end we present numerical results on more performant methods of the same type [2] for which no theory is available.

REFERENCES

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