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# A PBDW approach to Hamilton-Jacobi-Bellman equations

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The goal of this work is to use the Parametrized-Background Data-Weak (PBDW) method introduced in [3] for Hamilton-Jacobi-Bellman (HJB) equations to combine the solution of the HJB equations with given data.
The PBDW method is a variational data assimilation method for systems modeled by a parameterized partial differential equation. Unlike in [4], we use Reproducing Kernel Hilbert Spaces (RKHS) to process the point-wise given solutions of the HJB equations in addition to the point-wise noisy measurement data.
We use the PBDW method to combine two models of different complexity for an optimal trading problem in the context of intraday electricity markets with historical market data to obtain more realistic results, since the models cannot represent all electricity market phenomena. The more sophisticated model described in [1] can be seen as an extension of the model introduced in [2]. This extension leads to a higher computational cost, since the solution has to be computed numerically, unlike in the model from [2], where an analytical solution is available.
Of particular interest is whether the model with analytical solution assimilated by data can give as good results as the more sophisticated model.

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