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Casimir preserving numerical simulation of 2d homogeneous turbulence

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In the language of differential geometry, the two-dimensional dynamics of ideal fluids, in the Eulerian viewpoint, can be understood as a Lie-Poisson system [1]. This system has an infinte number of first integrals (called Casimirs), i.e., the integrated powers of vorticity. Numerical schemes that preserve the underlying differential structure of the problem are called geometric. The aim of this contribution will be to numerically analyse 2d turbulence, through simulation of the Navier-Stokes equations using geometric integration. In particular, we will be analysing the double cascade (inverse cascade of energy and direct cascade of enstrophy) of 2d turbulence and compare our numerical findings with those from reference pseudo-spectral methods [5].

Integration of Lie-Poisson PDE's requires two main steps. First, a finite-dimensional truncation of the corresponding Poisson bracket [2, 3] is introduced. Secondly, the design of a Lie-Poisson integrator, i.e., a numerical method that respects the Casimirs and is a Poisson map. While geometric integrators often rely on the computation of the exponential map, which becomes computationally prohibitive for fluid flows of physical relevance, it has been recently shown that there exist fast algorithms that exploit isospectrality of the flow [4]. The latter method will be employed and extended to efficiently work on large-scale computing facilities in order to enable genuinely large-scale simulations of turbulence.

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