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D2Q9 model of upwind lattice Boltzmann scheme for hyperbolic scalar conservation laws

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Lattice Boltzmann Method (LBM) is a popular kinetic theory based mesoscopic incompressible Navier-Stokes solver. Its popularity is due to the algorithmic simplicity involving streaming and collision operators. Development of LBM for compressible flow problems, with dominant hyperbolic convection terms, is non-trivial and is an active area of research. A novel LBM which is equivalent to an upwind method at macroscopic level is introduced in [1] for hyperbolic scalar conservation laws. While the equivalence of that Lattice Boltzmann (LB) algorithm with macroscopic Computational Fluid Dynamics (CFD) algorithms is an advantage, it is based on a simple *D2Q5-plus* model. In this work, the upwind LBM in [1] is extended to *D2Q9* model. A novel modification of the flux decomposition method of [2] is introduced for defining the equilibrium distribution functions to facilitate the use of *D2Q9* model. A specific form of rest particle component ensures the correctness of moment relations. An appropriate selection of split fluxes within this format results in upwinding even along diagonal directions. The *D2Q9* model results in a multi-faceted LBM, as the scheme varies based on the way the total flux gets decomposed between coordinate and diagonal-to-coordinate (45° from coordinate) directions, thereby becoming the superset of *D2Q5-plus* and *D2Q5-cross* models. A test problem from [3], involving multi-directional discontinuities, has been solved using the *D2Q9* model of upwind LBM. It is observed that the scheme captures the 45° discontinuity exactly for a specific partition of total flux between coordinate and diagonal-to-coordinate directions. The LB algorithm is tested further on standard problems for scalar conservation laws.

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