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Well-balanced high-order schemes for hyperbolic systems with stiff relaxation

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The aim of this work is to design high-order numerical methods for hyperbolic systems with relaxation. In the asymptotic limit, i.e., when the relaxation parameter ε is very small, these systems converge to others that could be of another type from the original one. For instance, a parabolic behaviour could be obtained in the asymptotic limit, as in the case of the hyperbolic heat equation. When ε is small, the relaxation term becomes very strong and highly stiff, and numerical schemes may produce spurious results. Implicit-explicit (IMEX) Runge–Kutta schemes have been widely used for the time evolution of hyperbolic partial differential equations (see [1]-[2]). Moreover, our goal is to recover an exactly well-balanced scheme for the asymptotic limit system, following the technique introduced by two of the authors, which is based on a well-balanced reconstruction operator (see [3]). Special care is put to approximate the averages and integral of the source terms using quadrature formulas. Some numerical experiments will be presented in order to check the efficiency of the methods and the well-balanced property.

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