

A Scalable Algorithm for Geometric Constrained Shape Optimization in Banach Spaces

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We present a PDE-constrained shape optimization methodology in fluid dynamic applications that preserves geometric constraints, specifically volume and barycenter, while allowing for large deformations. Special attention is placed on problems where domain singularities have to be removed and created as part of the optimization process. The presence of discretization-independent geometric constraints leads to a computationally challenging treatment of the optimization problem. An Augmented Lagrangian-based approach is presented in [3], where a nonlinear extension equation, which has been introduced in [2], can be used to promote deformations along large gradients. This leads to a highly parameter-dependent formulation, that has to be heuristically fine-tuned. A parameter-free approach is based on the p -Laplace operator, based on the work described in [1]. For this the geometric constraints are treated as part of the equation system, thus avoiding penalty terms. In addition, great advantages are taken from operating within a suitable Banach space compared to the previous approach, where the descent direction belongs to a Hilbert space. As a benchmark example, the boundary of an obstacle in the center of a flow tunnel is optimized for the aerodynamic drag. The obstacle is defined as a Lipschitz domain and the flow is described by Navier-Stokes equations. The methodology is tested for high number of degrees of freedom, for which the use of a large, distributed-memory system is necessary. Results for the scalability and grid-independence studies for the algorithms are shown as a first step towards industrial applications.

REFERENCES

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