The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

FINITE ELEMENT EXTERIOR CALCULUS APPLIED TO INCOMPRESSIBLE NAVIER-STOKES EQUATIONS.

MARIEN-LORENZO HANOT[1]

[1] IMAG, Université de Montpellier, CNRS, Montpellier, France,
Place Eugène Bataillon – Bat 9, 34095 Montpellier, France,
marien-lorenzo.hanot@umontpellier.fr

Keywords: *Finite Element, Exterior Calculus, de Rham Complex, incompressible Navier-Stokes*

We study the application of discrete de Rham complex to incompressible fluid equations in three dimensions. The discrete de Rham complex we use is based upon exterior calculus and was introduced by D. Arnold [4]. We devise a new formulation and scheme to solve the Navier-Stokes equations that preserve desirable properties: mainly the solutions are pointwise divergence free and the scheme is pressure-robust ([3]). Our scheme works for arbitrary order and keeps a low number of unknowns at low order. The use of rather common elements (i.e. belonging to the periodic table [2]) allows for an easy implementation using e.g. the FEniCS computing platform [1].

We prove the well-posedness as well as an error estimate in space for the linearized equations. We validate our results numerically and show some interesting perspectives, especially for rotating fluids for which we found convincing prediction of their qualitative behavior.

References

[1] A. Logg, K.-T. Mardal, and G. N. Wells. *Automated Solution of Differential Equations by the Finite Element Method.* Springer-Verlag, Berlin, 2012.

[2] D. N. Arnold and A. Logg. "Periodic table of the Finite Elements." In: *SIAM News* 47.9 (2014).

[3] V. John et al. "On the Divergence Constraint in Mixed Finite Element Methods for Incompressible Flows". In: *SIAM Rev.* 59.3 (2017), pp. 492–544. DOI: 10.1137/15m1047696.

[4] D. N. Arnold. *Finite Element Exterior Calculus*. Vol. 93. CBMS-NSF Regional Conference Series in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2018, pp. xii+120.