A posteriori error estimates by weakly symmetric stress reconstruction for the Biot problem

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A posteriori error estimates are constructed for the three-field variational formulation of the Biot problem involving the displacements, the total pressure and the fluid pressure. The discretization under focus is the $H^1(\Omega)$-conforming Taylor–Hood finite element combination, consisting of polynomial degrees $k + 1$ for the displacements and the fluid pressure and k for the total pressure. An a posteriori error estimator is derived on the basis of $H(div)$-conforming reconstructions of the stress and flux approximations. The symmetry of the reconstructed stress is allowed to be satisfied only weakly. The reconstructions can be performed locally on a set of vertex patches and lead to a guaranteed upper bound for the error with a constant that depends only on local constants associated with the patches and thus on the shape regularity of the triangulation. Particular emphasis is given to nearly incompressible materials and the error estimates hold uniformly in the incompressible limit. Numerical results on the L-shaped domain confirm the theory and the suitable use of the error estimator in adaptive strategies.

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