

## LEARNING AN OPTIMAL FEEDBACK OPERATOR SEMIGLOBALLY STABILIZING SEMILINEAR PARABOLIC EQUATIONS

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Stabilizing feedback operators are presented which depend only on the orthogonal projection of the state onto the finite-dimensional control space. A class of monotone feedback operators mapping the finite-dimensional control space into itself is considered. The special case of the scaled identity operator is included.

Conditions are given on the set of actuators and on the magnitude of the monotonicity, which guarantee the semiglobal stabilizing property of the feedback for a class semilinear parabolic-like equations.

Subsequently an optimal feedback control minimizing the quadratic energy cost is computed by a deep neural network, exploiting the fact that the feedback depends only on a finite dimensional component of the state. Numerical simulations demonstrate the stabilizing performance of explicitly scaled orthogonal projection feedbacks, and of deep neural network feedbacks.

## REFERENCES

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