

Well-balanced methods for one-dimensional blood flow model with discontinuous mechanical and geometrical properties

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We are interested in the numerical study of the one-dimensional blood flow model with discontinuous mechanical and geometrical properties proposed in [1]. In [2] this model was revisited, completing the mathematical analysis. Together with this analytical study, well-balanced schemes based on the ones in [3] were designed. They were able to preserve only *blood-at-rest* stationary solutions.

Our purpose in this work is to develop first- and second-order fully well-balanced numerical methods that are able to preserve every stationary solutions, not only the *blood-at-rest* ones. In order to do this we rewrite the system in a new compact form and we apply the well-balanced methodology developed in [4]. Several tests are shown in order to compare the schemes in [2] and the new ones. We observe an improvement in the convergence to the exact solution, highlighting the importance of the fully well-balanced property.

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