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Mixed Isogeometric Analysis for Linear Elasticity

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The theory of Finite Element Exterior Calculus (FEEC) is an elegant framework for the design of stable and convergent discretization schemes for different PDE problems. The exploitation of the underlying concepts involving Hilbert complexes and corresponding Hodge-Laplacians has been studied successfully in various application fields. The abstract formulation of FEEC makes it useful for different discretization approaches. Although FEM based on triangulations can be seen as the original discretization ansatz, Isogeometric Anlaysis (IGA) methods utilizing the FEEC results have been established, too. For example, Buffa et al. [2] developed a structure-preserving discretization of the de Rham complex using B-spline spaces that form an exact subcomplex of the continuous pendant.

Here we want to propose a method for planar linear elasticity following the idea of IGA discretizations for the de Rham complex. The basis for our scheme is the mixed three-field formulation with weak symmetry, i.e. the stress tensor symmetry is enforced by means of a Lagrange multiplier; see [1].

As a result we obtain a method which is stable for the nearly incompressible case and that is capable of arbitrary high convergence orders. Furthermore, due to low regularity requirements concerning the discrete spaces, the ansatz is suitable for multi-patch parametrizations and thus applicable for a variety of geometries. Numerical tests carried out utilizing MATLAB together with the GeoPDEs [3] package underline our theoretical findings.

Despite that we so far have studied the planar case only, we currently work on a generalization for three-dimensional solids. The connection between the de Rham sequence and the elasticity complex as outlined in [1] gives rise to the feasibility of such a step.

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