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Potential reconstruction techniques for *a posteriori* error estimation: a guided tour

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Consider the pure diffusion model defined in a domain $\Omega \in \mathbb{R}^n$ ($n = \{2,3\}$), and let $p \in H_0^1(\Omega)$ and $\mathbf{u} \in \mathbf{H}(\mathrm{div},\Omega)$ refer respectively to the exact potential and flux (in a weak sense), related via the diffusive law $\mathbf{u} = -\mathbf{S}\nabla p$, with $\mathbf{S}$ representing a symmetric, bounded, and positive-definite second-order tensor. In this investigation, we focus on upper bounds of the type

$$|||p - q||| \leq ||\mathbf{S}^{-1/2}\mathbf{v} - \mathbf{S}^{1/2}\nabla q||_{L^2(\Omega)} \qquad \forall\, q \in H_0^1(\Omega),\, \mathbf{v} \in \mathbf{H}(\mathrm{div},\Omega), \tag{1}$$

where $q \in H_0^1(\Omega)$ and $\mathbf{v} \in \mathbf{H}(\mathrm{div},\Omega)$ are *arbitrary* approximations to p and $\mathbf{u}$, respectively.

Naturally, one does not use arbitrary approximations, but rather solutions arising from numerical methods. In this context, if mixed dual approximations to the model are available, one counts with $\mathbf{u}_h \in \mathbf{H}(\mathrm{div},\Omega)$ and $p_h \in L^2(\Omega)$. Thus, it is necessary to increase the regularity of $p_h \in L^2(\Omega)$ and obtain a reconstructed potential $\tilde{p}_h \in H_0^1(\Omega)$.

This reconstruction process, however, is not unique, and since we only need *a* potential in $H_0^1(\Omega)$, there is some room for flexibility. This work aims at testing the performance of inteporlators of the type $\mathcal{G} : L^2(\Omega) \to H_0^1(\Omega)$, such that $\tilde{p}_h = \mathcal{G}(p_h)$, and (1) can be computed using $\tilde{p}_h \in H_0^1(\Omega)$ and $\mathbf{u}_h \in \mathbf{H}(\mathrm{div},\Omega)$. Our study encompasses reconstruction techniques used in [1], [2], and [3]. Numerical experiments are applied to the Darcy equation in two and three dimensions both for fractured and unfractured domains.

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