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Coxeter triangulations, their quality, and an efficient data structure

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This presentation will be based on [1, 2]. The first part of this talk will give an overview of Coxeter triangulations of $\mathbb{R}^d$.

Definition 1 Coxeter triangulations *are hyperplane arrangements* $\mathcal{H}_E = \{x \in \mathbb{R}^d \mid \langle x, u\rangle = k, u \in E, k \in \mathbb{Z}\}$ *(depending on the set of vectors -called roots- E) whose cells are d-dimensional simplices with a special property that two adjacent d-dimensional simplices in a Coxeter triangulation are orthogonal reflections of one another.*

All Coxeter triangulations have very good quality. By good quality we mean that the ratio of volume of the simplex to the longest edge length to the power of the dimension is large. It turns out that one particular family of Coxeter triangulations, namely those of type $\tilde{A}_d$ is particularly nice. Let H be the hyperplane of $\mathbb{R}^{d+1}$ of equation $\langle x, \mathbf{1}\rangle = 0$ where $\mathbf{1}$ is the vector of $\mathbb{R}^{d+1}$ whose coordinates are all 1. The set of roots that define the Coxeter triangulation of type $\tilde{A}_d$ in $H \simeq \mathbb{R}^d$ is

$$E_C = \{r_{i,j} = e_i - e_{j+1} | \ 1 \leq i \leq j \leq d\},$$

where e_i denotes a basis vector.

In the second part we therefore focus on this family and its relative the Freudenthal-Kuhn triangulation. We discuss a data structure for these triangulations that allows to locate a point and to retrieve the faces or the cofaces of a simplex of any dimension in an output sensitive way. The data structure has been implemented. The data structure is instrumental in our triangulation algorithm [2].

REFERENCES

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