

Homogenization of higher-order continua

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Following the well known technique FE²-method for homogenisation as shown in Keip et al. [2] of complex materials with micro-heterogenous properties, we present a novel IGA²-method, presented in Schmidt et al. [1], allowing the homogenization of higher-order continua. To be specific, we show a generalization of the work of Kouznetsova [4] on higher-order material models towards second and third order continua on both scales.

Investigating the microscopic structure we make use of representative volume elements (\mathcal{RVE}), attached at every macroscopic material point. An energetic criterion on both scales is applied to derive suitable boundary conditions on $\partial\mathcal{RVE}$. For the discretization the IGA²-method is introduced, adapting the principles of the FE²-method. Furthermore, macroscopic stresses and their consistent linearization for the Newton-Raphson iteration are obtained. Eventually, the accuracy and behavior of the introduced framework for the homogenization of higher-order continua is demonstrated in a variety of benchmark tests. Here, we make use of second order formulations on fibre reinforced materials as introduced in Dittmann et al. [3] and show a simple multigrid solution scheme.

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