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ANALYSIS OF NONSMOOTH MULTIGRID FOR PHASEFIELD BRITTLE FRACTURE

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Many phasefield formulations of fracture formation are written as flows of an energy functional with a rate-independent dissipation term. Increment problems of such flows are typically nonsmooth minimization problems, for energies that are convex in each variable, but not convex overall. Such problems are typically solved using operator-splitting or monolithic Newton-type methods. These are expensive, because they solve sequences of linear problems. Also, their convergence behavior is not always clear.

In this talk we consider exemplarily the brittle-fracture phasefield model of Miehe et al. [2], with an energy split according to the eigenvalues of the elastic strain. We prove a number of convexity and regularity results for the increment energy, using the theory of spectral functions. This allows to apply a nonsmooth multigrid (TNNMG) solver to the increment problem, and to prove its global convergence for any initial iterate [1].

We then show the numerical performance of the multigrid solver. In a number of experiments involving both the Ambrosio–Tortorelli 1 and 2 models it outperforms state-of-the-art operator-splitting solvers by an order of magnitude [3].

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