

## Reservoir computing of three-dimensional turbulent convection

Florian Heyder<sup>1</sup> and Jörg Schumacher<sup>1,2</sup>

<sup>1</sup> Technische Universität Ilmenau, D-98684 Ilmenau, Germany,  
florian.heyder@tu-ilmenau.de

<sup>2</sup> New York University, New York City, NY 11201, USA

**Keywords:** *Direct Numerical Simulations, Convolutional Autoencoder, Reservoir Computing, Thermal Convection*

Turbulent convection forms a central process in Earth's climate system. As the corresponding length and time scales range over many orders of magnitude, general circulation models compute the large-scale flow only, while small-scale turbulence has to be parameterized. A typical parameterization scheme consists of a local direct numerical simulation (DNS) which interacts with the large scale flow via its long-term statistics. Even though successful, such a procedure demands additional high computational costs for solving the non-linear equations of fluid motion at the small scales.

We propose an alternative approach with an echo state network (ESN) for time series forecast. The ESN is one implementation of a recurrent neural network, where only the output layers are trained by linear regression. This reservoir computing (RC) method serves as light weight substitute for a direct solution of the underlying fluid equations, as the training process is computationally inexpensive. RC has shown great promise in predicting two-dimensional convection flow with correct statistical properties [1]. Recently it has been demonstrated that it can also generalize between different boundary conditions and infer correct profiles of a dry atmospheric boundary layer model [2].

Here we train a convolutional autoencoder network on three-dimensional DNS data of Rayleigh-Bénard convection. The encoder part allows us to compress the temperature  $T$  and velocity fields  $u_x, u_y, u_z$  to a low-dimensional latent space of dimension of approximately  $\sim 10^2$ . This low-order representation is then fed to an ESN in order to perform the task of prediction of dynamics. The advanced latent space is then processed by the decoder of the network to yield a prediction to the three-dimensional flow. We analyze the predicted three-dimensional flow in terms of its large-scale patterns and statistical properties. Our results indicate that this approach can be used as low-cost parameterization scheme in large-scale circulation models.

## REFERENCES

- [1] F. Heyder and J. Schumacher, Echo state network for two-dimensional turbulent moist Rayleigh-Bénard convection. *Phys. Rev. E*, Vol. **103**, article no. 053107, 2021
- [2] F. Heyder, Juan Pedro Mellado and J. Schumacher, Two-dimensional convective boundary layer: Numerical analysis and echo state network model. *arXiv:2108.06195*, 2021.