

Structure-preserving discretizations and preconditioning for incompressible MHD models

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Keywords: *Magnetohydrodynamics, Preconditioners, Structure-preserving, Finite Elements*

The incompressible magnetohydrodynamics equations are characterised by several conservation laws such as the magnetic Gauss's law and the conservation of energy and magnetic and cross helicity in the ideal limit.

In the first part of this talk, we give an overview of structure-preserving finite element discretizations that enforce these laws on the discrete level up to machine precision and solver tolerance.

We proceed by presenting a scalable augmented Lagrangian preconditioner that achieves good performance at high Reynolds and coupling numbers. Our approach relies on specialized parameter-robust multigrid methods for the hydrodynamic and electromagnetic blocks. We briefly explain the importance of structure-preserving discretizations for the construction of these methods.

We finish with a remark on how the presented methods can be extended to the Hall MHD equations.