

## Refinement Strategies for Locally Linearly Independent LR B-splines

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Locally Refined (LR) B-splines have been introduced in [2] as generalization of the tensor product B-splines to achieve adaptivity in the discretization process. Thereby, the approximation efficiency is dramatically improved as one avoids the wasting of degrees of freedom by increasing the number of basis functions only where rapid and large variations occur in the analyzed object. Nevertheless, the adoption of LR B-splines for simulation purposes, in the Isogeometric Analysis (IgA) framework, is hindered by the risk of linear dependence relations [4]. Although a complete characterization of linear independence is still not available, the *local* linear independence of the basis functions is guaranteed when the underlying Locally Refined (LR) mesh has the so-called Non-Nested-Support (N<sub>2</sub>S) property [1]. The local linear independence not only avoids the hurdles of dealing with singular linear systems, but it also improves the sparsity of the matrices when assembling the numerical solution. Such a strong property of the basis functions is a rarity, or at least it is quite onerous to gain, among the technologies used for adaptive IgA.

To the best of my knowledge, only three refinement strategies have been proposed to build LR meshes with the N<sub>2</sub>S property so far: the Non-Nested-Support-Structured (N<sub>2</sub>S<sub>2</sub>) mesh refinement [5], the Effective Grading (EG) refinement [3] and the Hierarchically Locally Refined (HLR) mesh refinement [1]. In this talk I provide an overview of them, highlighting and comparing their properties.

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