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3D-1D coupled problems with a PDE-constrained optimization method

S. Berrone[1], D. Grappein[2], S. Scialó[3] and F. Vicini[4,*]

[1] Dipartimento di Scienze Matematiche, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, 10129, Italy, stefano.berrone@polito.it
[2] Dipartimento di Scienze Matematiche, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, 10129, Italy, denise.grappein@polito.it
[3] Dipartimento di Scienze Matematiche, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, 10129, Italy, stefano.scialo@polito.it
[4] Dipartimento di Scienze Matematiche, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, 10129, Italy, fabio.vicini@polito.it
[*] Presenter

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Nowadays, coupled 3D-1D problems are present in many applications, such as roots-soil interaction, geological porous media with artificial wells simulations or interactivity between the vascular system and the biological tissues. The numerical analysis of those problems is known to be non-standard, as stated for example in [1], because a bounded trace operator definition is missing when the dimensionality gap is higher than one. We propose a well posed 3D-1D formulation obtained by reducing the 3D-3D problem with thin inclusions. The 3D and 1D solutions are obtained through a PDE-constrained optimization problem, in which a cost functional expressing the interface conditions is minimized constrained to the physical elliptic equations. A preconditioned conjugate gradient strategy is presented with a non-conforming mesh schema at 3D-1D interfaces to tackle large scale problems efficiently. Finite Elements (FEM) are used, but the approach can be extended properly with other numerical methods. The results presented and further details can be found in [2].

REFERENCES

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