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Comparative Study on a Variety of Structure-Preserving High Order Spatial Discretizations with Entropy Split Methods for MHD

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Sjögreen and Yee [11, 12] proved that the high order entropy split methods based on Harten's entropy function [4] are entropy stable for central differencing with summation-by-parts (SBP) operators for both periodic and non-periodic boundary conditions for nonlinear thermally-perfect gas dynamics Euler equations. The objective of this paper is to use the same Harten entropy function for the ideal MHD governing equation set. Test cases are included to compare with a variety of structure-preserving high order spatial discretizations. Here structure-preserving numerical methods pertain to physical-preserving methods for spatial discretizations with accurate time-discretizations using fine enough time steps. In particular, the comparative studies concentrate on Tadmor-type of discrete entropy conserving [14], momentum conserving [2], kinetic energy preserving [5, 6, 1], Yee et al., Sjögreen & Yee [15, 8, 10, 11, 9, 12] entropy split methods and their combination of these physical-preserving methods [7]. All of these considered methods are on their own right preserving certain physical properties of the chosen governing equations but also known to either improve numerical stability, and/or minimize aliasing errors in long time integration in turbulent flow simulations without the aid of added numerical dissipation for selected flow types [9, 3].

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