

Goal-Oriented hp -Adaptive Finite Element Methods: A Painless Multilevel Automatic Coarsening Strategy For Non-SPD Problems

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The global energy of a problem may be a quantity of limited relevance in many engineering applications. This limitation motivated the design of goal-oriented adaptive finite element methods (see, e.g. [1-3]). They intend to approximate a particular Quantity of Interest (QoI) using an error estimator based on the solution of the adjoint problem to guide the refinements.

In the context of hp -adaptivity, Darrigrand et al. [4] proposed an algorithm for symmetric and positive definite (SPD) problems based on performing global refinements followed by optimal unrefinements. This algorithm marks the basis functions with the lowest contributions to the energy of the solution and removes them. The resulting automatic hp -adaptive strategy employs a multi-level hierarchical data structure proposed by Zander et al. [5].

In this presentation, we extended Darrigrand et al. [4] algorithm to SPD and non-SPD problems in the framework of goal-oriented adaptivity. To do so, we re-define and evaluate the contribution to the energy considering both the direct and the adjoint problem. As a result, we obtain a hp -adaptive algorithm for SPD and non-SPD problems in the context of goal-oriented adaptivity. We test and analyze our algorithm on two-dimensional (2D) Poisson and Helmholtz problems, and we describe the main features and limitations of the proposed method. In particular, our algorithm is robust and straightforward to implement; therefore, it can be used for industrial applications.

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