The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

On quadrature rules for solving Partial Differential Equations with Neural Networks

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Keywords: *Deep Learning, Neural Networks, Ritz method, Quadrature rules*

Neural Networks have been widely used to solve Partial Differential Equations (PDEs). Deep Neural Network (DNN) techniques present several advantages with respect to traditional PDE solvers based on Finite Elements, Finite Differences, or Isogeometric Analysis. Among the advantages of DNNs, we encounter the nonnecessity of generating a grid. In general, DNNs use a dataset in which each datum is independent from others. Furthermore, DNNs provide the possibility of solving certain problems that cannot be solved via traditional methods, like high-dimensional PDEs, fractional PDEs, and multiple nonlinear PDEs.

When solving PDEs with NNs, definite integrals are required to be solved. In [1], we illustrate via 1D numerical examples the quadrature problems that may arise in these applications and propose different alternatives to overcome them, namely: Monte Carlo methods, adaptive integration, piecewise-polynomial approximations of the Neural Network output, and the inclusion of regularization terms in the loss. We also discuss the advantages and limitations of each proposed alternative. In high dimensions (above three or four), we recommend the use of Monte Carlo integration methods [2]. In lower dimensions (three or below), we suggest the use of adaptive integration or polynomial approximations. The use of regularization terms is a mathematically elegant alternative that is valid for any spatial dimension, however, it requires certain regularity assumptions on the solution and complex mathematical analysis when dealing with sophisticated Neural Networks.

REFERENCES

[1] Rivera J.A., Taylor J.M., Omella A.J. and Pardo D. On quadrature rules for solving Partial Differential Equations using Neural Networks. arXiv preprint arXiv:2111.00217, (2021).

[2] Weinan E. and Bing Yu. The Deep Ritz Method: A Deep Learning-Based Numerical Algorithm for Solving Variational Problems. *Communications in Mathematics and Statistics*, (2018) **6**: 1–12.