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# A Generative Adversarial Networks approach for solving Partial Differential Equations

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Solving Partial Differential Equations (PDEs) has been a long-standing challenge in numerical analysis and computation. Traditional numerical methods have been widely developed and studied in the last decades, and strong contributions have been found about their convergence. For example, in

$$\begin{cases} \text{Find } u^* \in U \text{ such that} \\ b(u^*, v) = l(v), \ \forall v \in V, \end{cases} \tag{1}$$

the following optimization process solves the problem:

$$u^* = \arg\min_{u\in U} \sup_{0\neq v\in V} \frac{|b(u,v) - l(v)|}{||v||_V}, \tag{2}$$

where $U$ and $V$ are the trial and test spaces, respectively, $b$ is a bilinear form, $l$ is a linear functional, and $||\cdot||_V$ is a norm induced in $V$.

Neural Networks have demonstrated their great power in recent years on solving PDEs. In particular, [1] proposes to solve (2) for a weak formulation setting. For that, the authors combine two networks adversarially: one to approximate the trial solution, and another one for approximating the test maximizer; however, they select suboptimal norms for the optimization.

In our work, we revisit the Weak Adversarial Networks and we make an extensive review of the related theory to improve their problem setting and we propose enhanced optimization strategies. Moreover, we extend their proposal to a wider class of problems and formulations, including strong and ultra-weak formulations.

## REFERENCES

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