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Multilevel Delayed Acceptance MCMC: Cascading Distributions, Variance Reduction and Adaptive Error Models

Mikkel B. Lykkegaard[1,*], Tim J. Dodwell[2], Colin Fox[3], Grigorios Mingas[4] and Robert Scheichl[5]

[1]IDSAI, University of Exeter, UK, `m.lykkegaard@exeter.ac.uk`
[2]The Alan Turing Institute and IDSAI, Uni. of Exeter, UK, `t.dodwell@exeter.ac.uk`
[3]Dept. of Physics, University of Otago, New Zealand, `colin.fox@otago.ac.nz`
[4]The Alan Turing Institute, London NW1 2DB, UK, `gmingas@turing.ac.uk`
[5]IWR, Heidelberg University, Germany, `r.scheichl@uni-heidelberg.de`

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We present a novel MCMC algorithm, titled Multilevel Delayed Acceptance (MLDA). The algorithm is capable of sampling from the exact target distribution using a hierarchy of distributions of increasing complexity and computational cost and can be considered as an amalgam of two existing methods, namely the Delayed Acceptance (DA) MCMC of Christen and Fox [1] and the Multilevel MCMC (MLMCMC) of Dodwell *et al.* [2].

The original DA algorithm was designed to use a single coarse distribution to filter MCMC proposals before computing the fine density. We extend this approach in two ways. *Vertically*, by allowing any number of coarse distributions to underpin the target and *horizontally*, by allowing the coarse level samplers to generate extended subchains of either fixed or random lengths. The resulting algorithm is in detailed balance with the exact target distribution. We show that MLDA can be exploited for variance reduction, similarly to MLMCMC, and construct a multilevel error model that adaptively aligns the coarse distributions to the target with little additional computational cost.

The MCMC samples generated by MLDA are indeed Markovian, unlike MLMCMC, where detailed balance is theoretically only ensured at infinite computational cost. However, MLDA is strictly sequential and hence suffers from resistance to parallelisation, like most MCMC algorithms. It also introduces an additional tuning parameter, namely the subsampling length for the coarse samplers. We discuss opportunities for parallelising and tuning MLDA using concepts from reinforcement learning.

REFERENCES

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