

# Numerical model reduction using POD and spectral decomposition for computational homogenization of porous media

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Computational homogenization can be used in order to model the effective mechanical behavior of fluid saturated porous rock with heterogeneous properties. A standard approach is the “finite element squared” (FE<sup>2</sup>) procedure, where a new boundary value problem for the coupled porous media problem is defined on Representative Volume Elements (RVE) in each quadrature point of the (macroscale) mesh. The effective macroscopic response is obtained from solving the RVE problem, which takes the place of a classical material model. For fine macroscale meshes, The FE<sup>2</sup> strategy can be computationally expensive which is why it is of interest to reduce the cost of solving the individual RVE problems by introducing a reduced basis, here denoted Numerical Model Reduction (NMR).

Jänicke et al. [1] used proper orthogonal decomposition (POD) to find a reduced basis for homogenization of quasi-static linear poroelasticity and it was demonstrated that the apparent macroscale properties pertain to viscoelasticity. Naturally the richness of the reduced basis will determine the accuracy of the (reduced) solution, which calls for error control. To approximate the error stemming NMR, Ekre et al. [2] developed an *a posteriori* error estimator for estimation of the error in (i) energy norm, and (ii) in arbitrary quantities of interest. From the structure of the estimator it was also found that using spectral modes, in combination with POD modes, can be advantageous in order to obtain a shaper estimate. In this contribution we present the estimator, and demonstrate the effectiveness using numerical examples.

## REFERENCES

- [1] R. Jänicke, F. Larsson, K. Runesson, and H. Steeb. Numerical identification of a viscoelastic substitute model for heterogeneous poroelastic media by a reduced order homogenization approach. *Comput. Methods Appl. Mech. Engrg.* (2020), DOI: 10.1016/j.cma.2015.09.024.
- [2] F. Ekre, F. Larsson, K. Runesson, and R. Jänicke. A posteriori error estimation for numerical model reduction in computational homogenization of porous media. *Int J Numer Methods Eng.* (2020), DOI: 10.1002/nme.6504.