Acoustic and elastic wave propagation in microstructured media with interfaces: homogenization, simulation and optimization

In this thesis, the focus is on wave propagation in periodic microstructured media in the presence of interfaces. The dynamic homogenization of these media and the design of the microstructures to achieve a given macroscopic effect are studied. In a first part, homogenization and optimization are carried out for thin microstructured layers. In a second part, the homogenization of periodic microstructures in all spatial dimensions is addressed.

The first part concerns the case where the heterogeneities constitute a periodic row of inclusions immersed in a homogeneous matrix. When the physical parameters of the inclusions are strongly contrasted with those of the matrix, internal resonances can occur and be used to maximise acoustic absorption. The homogenization of such a resonant microstructured layer is studied using a method of matched asymptotic expansions and leads to non-local jump conditions. The introduction of auxiliary variables allows to get a local evolution problem in time which is then solved numerically by an ADER scheme coupled with an immersed interface method. This methodology is validated (local truncation error analysis, comparison with analytical solutions) and makes possible wave diffraction simulations by resonant meta-interfaces. Finally, the sensitivity of the effective parameters to the geometry of the microstructure is determined using topological derivatives. We then implement a topological optimization procedure for the design of non-resonant thin microstructured layers.

On the other hand, it is often assumed that the contact between the inclusions and the homogeneous matrix is perfect. Some models, such as spring-mass conditions, account for the behaviour of imperfect contacts between solids. In the second part of the thesis, low-frequency volume homogenization of such configurations is carried out to obtain the expression of the homogenized fields at order 1, and an extension to non-linear contacts is presented. Finally, dispersion diagrams in 1D solids with spring-mass conditions are studied. The framework of high-frequency homogenization is used and an approximation of the fields to the leading order, as well as dispersion relations near the edges of the Brillouin zone is obtained