

LINEAR AND NONLINEAR 1D-3D MODELS FOR FLOW AND TRANSPORT IN POROUS MEDIA WITH EMBEDDED TUBULAR NETWORKS

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Mixed-dimension 1D-3D methods allow to efficiently describe systems tubular networks embedded in porous media where fluid is exchanged between network and the porous bulk and dominates the fluid distribution in the porous medium. Two examples of such systems are fluid flow in microvasculature and embedding tissue [1], and water uptake of plant roots from soil [2]. Due to the nature of thin inclusion fluid pressure profiles in the porous bulk usually exhibit large gradients in the vicinity of the inclusion and pose difficulties for numerical schemes approximating pressure profiles and fluid flow. For example, for root-soil interaction models, particularly for dry soils, local water pressure gradients can become very large in the vicinity of roots due to the nonlinear relationships of water saturation, water pressure and soil permeability. State-of-the art models require infeasibly strong local grid refinement to capture these gradients.

We present a numerical scheme that combines smooth distribution functions in the vicinity of the network with interface reconstruction schemes based on local analytical solutions. We explore accuracy and limitations of the scheme in numerical test cases. Moreover, we discuss issues regarding time-dependent problems as well as implementation aspects of the presented methods for a finite volume discretization.

REFERENCES

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