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Parallel Space-Time Multilevel Methods with Application to Electrophysiology

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We consider the space-time discretization of the anisotropic diffusion equation, using an isogeometric analysis (IgA) approximation in space and a discontinuous Galerkin (DG) approximation in time. Drawing inspiration from a former spectral analysis of space-time operators in [1], we propose for the resulting space-time linear system a parallel multigrid preconditioned GMRES method, which combines a preconditioned GMRES with a standard multigrid acting only in space. In case of complex geometries we consider a hierarchy of non-nested meshes, created by semi-geometric coarsening. This "grey box" multigrid starts from a single fine spatial mesh and automatically generates space-time coarse meshes of any dimension. We analyze the convergence and scaling behaviour of the proposed solution strategies, focusing on the spectral properties and conditioning of the underlying discrete operators that arise from the tensor space-time finite element discretization and the usage of parallel block preconditioners. The application of this multilevel space-time strategy to a non-linear reaction-diffusion problem (i.e. the monodomain equation) will be also discussed [2]. Strong and weak scaling of the multilevel space-time approach is compared to PFASST algorithm (Parallel Full Approximation Scheme in Space and Time), highlighting properties and conceptual and quantitative differences of both approaches [3].

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