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A geometric finite element method for MHD that preserves energy, cross-helicity, magnetic helicity, incompressibility, and div B = 0

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The governing equations for magnetohydrodynamics (MHD) possess a number of conserved quantities that are difficult to preserve in numerical discretizations. In recent work [1], we constructed a finite element method for inhomogeneous, incompressible MHD that preserves energy, cross-helicity (when the fluid density is constant), magnetic helicity, incompressibility, and $\operatorname{div} B = 0$ to machine precision. This talk will summarize the method and discuss extensions to compressible and resistive MHD [2].

To derive the method, we make use of the the variational formulation of fluid dynamics on diffeomorphism groups. In this formulation, the fluid motion is regarded as a diffeomorphism of the fluid domain that extremizes an action functional: the time-integral of the fluid's kinetic energy minus its potential energy. As shown in [3], one can discretize this variational principle to construct structure-preserving finite element methods for fluid flow. We did so in [3, 4] for fluids with variable density and more recently for MHD in [2, 1]. The focus of this talk will be on MHD, but techniques from [3, 4] will nonetheless play an important role.

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