

## DYNAMIC BEHAVIOUR AND STABILITY OF LOCALIZED BIPENALTY FORMULATION IN CONTACT-IMPACT PROBLEMS

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Often, the finite element method together with direct time integration is used for modelling of contact-impact problems of bodies. For direct time integration, the implicit or explicit time stepping are generally applied. It is well known that the time step size in explicit time integration is limited by the stability limit. Further, the trouble comes with the task of impact of bodies with different critical time step sizes for each body in contact. In this case, this numerical strategy based on explicit time stepping with same time step size for both bodies is not effective and is not accurate due to the dispersion behaviour and spurious stress oscillations. For that reason, a numerical methodology, which allows independent time stepping for each body with its time step size, is needed to develop.

In parallel, the classical penalty method for enforcement the impenetrability condition in contact mechanics produces the sensitivity of critical time step size on penalty stiffness parameter – with increasing the stiffness penalty parameter the critical time step size decreases. This behaviour is avoided by the bipenalty formulation, where the mass penalty term for enforcement of the gap rate for persistency condition is activated [3]. By this way, the critical time step size is not chanced with a proper choice of the ratio of penalty parameters. Also, the stabilization of the spurious oscillations of contact forces can be realized with the predictor-corrector method [2].

The partitioned analysis for impact problems based on localized Lagrange multipliers [1] allows totally separation/splitting of governing equations of motion and corresponding equations of motion with given boundary and initial conditions can be integrated with independent time step sizes. In this contribution, we suggest the combination of the bipenalty formulation of contact-impact problems together with the localized variant of Lagrange multipliers for correct solution of impact-contact problem. The governing equations with stability analysis for one-dimensional case is presented and discussed together with potential asynchronous time integration scheme.

### REFERENCES

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