

## IMPROVING THE FINITE-ELEMENT MODELLING OF STRAIN-GRADIENT MODELS

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Strain-gradient theories have been proposed for modelling the micromechanical behaviour of materials. Besides important open questions on applicability and on calibration of the required micromechanical constitutive parameters, numerical implementation issues still exist, with new finite element formulations still being introduced. It is well known that a displacement-only finite-element formulation for these models leads to the requirement for elements with  $C^1$  continuous interpolation, with several having successfully been proposed [1]. In search of simpler interpolations, alternative mixed formulations have been proposed which allow the use of more common element shape functions, by introducing for example Lagrange multipliers [2].

We critically review long-standing and recent results on the finite-element modelling of strain-gradient theories. We use a recently developed framework [3] to discuss how different mixed elements in the literature compare to each other and to existing  $C^1$  continuous formulations, and consider to what extent further gains can be expected in the future for each approach. We show that margins for improvement are limited in two-dimensions, but more interesting improvements can be sought in three dimensions. At the same time, we consider the effect that any improvements on computational cost can have on the accuracy of the results. A possible avenue for achieving better-performing elements is to develop mixed elements for strain-gradient theories that avoid the need for either Lagrange multipliers or penalty parameters. We present a novel element formulation following this idea together with a discussion of the drawbacks and benefits of the proposed approach.

## REFERENCES

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