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Towards entropy–stable finite element moment methods for the Boltzmann equation

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In this talk we develop an entropy-stable finite-element-moment method for the Boltzmann equation with binary collisions [1] in bounded domains. The proposed method engenders a discontinuous-Galerkin method in position and temporal dependence, and a moment-method in velocity dependence. We base our moment-method on a converging sequence of approximations to the binary collision operator $\mathcal{C}$, denoted by $\mathcal{C}_N$. We associate with each member of the sequence a φ-divergence-based renormalization-map and entropy [2]. We show that each $\mathcal{C}_N$ inherits salient properties from $\mathcal{C}$, such as the preservation of the collision invariants, Galilean invariance, and that the linearization of $\mathcal{C}_N$ coincides with the linearization of $\mathcal{C}$. We show that our proposed finite-element-moment method is entropy-stable for each member of the sequence of approximations to $\mathcal{C}$. Finally, we apply our finite-element-moment method to the Boltzmann equation with collision operator $\mathcal{C}$ and demonstrate the corresponding approximation properties, using benchmark test cases, in comparison to Direct Simulation Monte Carlo.

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