

A Cut Finite Element Method for Ionic Electrodifffusion Problems on Resolved Cell Geometries

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We present a cut finite element method (CutFEM) [1] for numerically solving the so-called EMI (Extracellular-Membrane-Intracellular) model, with the goal of developing more efficient simulation of electric activity in explicitly resolved brain cell geometries. The EMI model is an example of a mixed-dimensional problem which couples an elliptic partial differential equation (PDE) on the extra/intracellular domains with a system of nonlinear ordinary differential equations (ODE) over the cell membranes. Because of their complex geometry, generating volumetric meshes that conform to the brain cells is challenging. A way to remedy this problem is the cut finite element method, in which even complex cell network geometries can be represented independently of the background mesh. Our method departs from an earlier developed Godunov splitting scheme for the model [2], which decouples the PDE part from the ODE part. We introduce a novel CutFEM formulation for the PDE step. In addition, we introduce a stabilized mass matrix approach to discretize the ODE system posed on the unfitted membrane surface. Finally, we test the complete splitting scheme for the fully-coupled model, showing that solving the EMI model using a CutFEM formulation is a promising approach for more simple and flexible handling of complex cell geometries.

REFERENCES

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