The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Learning cut-cell integration by means of deep neural networks

Rene R. Hiemstra[1] **and Dominik Schillinger**[1]

[1] Institute of Mechanics, Computational Mechanics Group, Technical University of Darmstadt, Karolinenpl. 5, 64289 Darmstadt, Germany,
hiemstra@mechanik.tu-darmstadt.de, schillinger@mechanik.tu-darmstadt.de

Keywords: *Fictitious domain methods, Immersed boundary methods, Finite cell method, Numerical quadrature, Cut-cell integration, Deep learning, Supervised learning, Artificial neural networks*

The focus of this work is to simplify and accelerate numerical integration for fictitious domain methods [1]. This is achieved by replacing existing methods for cut-cell integration [2, 3], which are typically cumbersome to implement and computationally expensive, by means of certain trained artificial networks. For a specific cut-case, the networks predict a set of quadrature weights that correspond to an a-priori fixed, tensor-product stencil of quadrature points. This means that one stencil, e.g. the Gauss-Legendre points, can be used to compute integrals on cut-elements, which facilitates implementation and efficient evaluation based on sum-factorization. The objective function used to train the neural networks is based on moment-fitting of all tensor product polynomials of order q on a cut-element. We explain in detail our supervised learning approach, which includes the process of generating valid training data, the approach used to minimize the objective function, and validation and verification of the resulting quadrature rules. In particular, we investigate the relative error incurred by the training data and validation data, and study the accuracy and computational cost of the quadrature rules when applied in the context of the Galerkin finite element method. The presented approach exhibits significant speed-ups in quadrature and formation of finite element arrays and significantly simplifies the implementation of fictitious domain type of numerical methods.

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