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# An entropy–stable discontinuous Galerkin approximation of the Spalart–Allmaras turbulence model for the compressible Reynolds Averaged Navier–Stokes equations

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We present an entropy–stable formulation for the compressible Reynolds Averaged Navier–Stokes (RANS) Discontinuous Galerkin (DG) equations and the Spalart–Allmaras one–equation closure. The model is designed to satisfy an entropy law, which includes free– and no–slip wall boundary conditions. We then construct a high–order DG approximation of the model that satisfies the summation–by–parts simultaneous–approximation–term (SBP-SAT) property. With the help of a discrete stability analysis, we construct two approximations: a kinetic energy preserving scheme based on Pirozzoli's two–point flux and a thermodynamic entropy conserving one based on Chandrashekar's split–form. The schemes are applicable to, and the stability proofs hold for, three–dimensional unstructured meshes with curvilinear hexahedral elements. We test the convergence of the schemes on a manufactured solution for increasing polynomial orders and mesh refinement levels, to then assess their numerical stability by propagating a flow from a random initial condition, and finally solve the flow around a two–dimensional flat plate and a NACA 0012 airfoil, comparing numerical results with those available in the literature. The proposed schemes are entropy–stable, and provide accurate solutions for the selected test cases.

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