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Hamiltonian models of the macroscopic Maxwell equations: continuous and discrete

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The macroscopic Maxwell equations posses a highly general Hamiltonian formulation which accommodates even nonlinear polarizations and magnetizations. The Hamiltonian formalism provides a coherent modeling framework which is advantageous, among other reasons, for discovering conserved quantities, for prescribing a meaningful notion of energy, and for clearly elucidating connections between the myriad models used in plasma physics. Hamiltonian structure has recently been shown to be valuable in designing structure preserving discretizations in plasma physics. By considering the electromagnetic fields as differential forms, the Hamiltonian structure of the macroscopic Maxwell equations may be spatially discretized using finite element spaces which preserve the de Rham cohomology to yield a Hamiltonian system of ODEs which preserve the Gauss constraints as Casimir invariants. It is physically relevant to distinguish between orientation dependent twisted differential forms and straight forms in Maxwell's equations with the constitutive relations being associated with a Hodge star operator. The notion of Poincaré duality naturally leads to dual grid methods with a discrete Hodge star operator acting as the map between the two grids. On the other hand, casting the theory entirely in terms of a single de Rham complex with duality defined in terms of the L^2 inner product yields a method amenable to discretization using finite element exterior calculus. Because the spatially discretized system is a noncanonical Hamiltonian system, depending on the the Hamiltonian being studied, it is sometimes possible to construct Poisson integrators via Hamiltonian splitting methods which exactly preserve the Poisson structure during time-stepping.

REFERENCES

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