

## On Geometric PIC-like discretizations of Lie-Poisson brackets

William Barham<sup>1,3</sup>, Philip J. Morrison<sup>2,3</sup> and Eric Sonnendrücker<sup>4,5</sup>

<sup>1</sup> Oden Institute for Computational Engineering and Sciences, The University of Texas  
at Austin

201 E. 24th Street, POB 4.102, Mail code: C0200, Austin, Texas 78712-1229  
william.barham@utexas.edu

<sup>2</sup> Department of Physics, The University of Texas at Austin  
2515 Speedway, Austin, TX 78712  
morrison@physics.utexas.edu

<sup>3</sup> The Institute for Fusion Studies, The University of Texas at Austin  
2515 Speedway, Austin, TX 78712

<sup>4</sup> Max-Planck-Institut für Plasmaphysik, Numerische Methoden in der Plasmaphysik  
Boltzmannstraße 2, 85748 Garching, Deutschland  
Eric.Sonnendruecker@ipp.mpg.de

<sup>5</sup> Technische Universität München, Zentrum Mathematik  
Boltzmannstraße 3, 85748 Garching, Deutschland

**Keywords:** *Structure preserving discretization, Hamiltonian dynamics, Fluid/vortex dynamics*

Virtually all non-dissipative models in plasma physics, from the Liouville equations and the BBGKY hierarchy to various kinetic and fluid models, have been shown to possess a Lie-Poisson structure when modeled as noncanonical Hamiltonian systems. In discretizing such brackets, one encounters a closure problem. That is, given a finite representation of the fields, it is usually not the case that the dynamic evolution of those fields is prescribed only in terms of that finite dataset. Particle based representations circumvent this difficulty with relative ease, but typically suffer from limited accuracy and difficulties in coupling to grid-based variables. We consider a particle-in-cell type discretization for the 2D Euler equations which preserves the Hamiltonian structure at the discrete level while also offering some of the advantages of a grid based discretization. Moreover, the strategy holds promise for application to general Lie-Poisson brackets.