

On Geometric PIC-like discretizations of Lie-Poisson brackets

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Keywords: *Structure preserving discretization, Hamiltonian dynamics, Fluid/vortex dynamics*

Virtually all non-dissipative models in plasma physics, from the Liouville equations and the BBGKY hierarchy to various kinetic and fluid models, have been shown to possess a Lie-Poisson structure when modeled as noncanonical Hamiltonian systems. In discretizing such brackets, one encounters a closure problem. That is, given a finite representation of the fields, it is usually not the case that the dynamic evolution of those fields is prescribed only in terms of that finite dataset. Particle based representations circumvent this difficulty with relative ease, but typically suffer from limited accuracy and difficulties in coupling to grid-based variables. We consider a particle-in-cell type discretization for the 2D Euler equations which preserves the Hamiltonian structure at the discrete level while also offering some of the advantages of a grid based discretization. Moreover, the strategy holds promise for application to general Lie-Poisson brackets.