

## Efficient geometrically exact formulation for curved beams

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In this work, an efficient two-noded curved beam element formulation is proposed. It extends the straight-beam model proposed in [1], properly accounting for geometrically nonlinear effects in a beam that can have an arbitrary initial shape. Cross sections are assumed to remain planar and perpendicular to the deformed beam axis. The formulation exploits the equilibrium equations in their integrated form (derived by integrating the differential equations of equilibrium, or simply from free-body diagrams), combined with the kinematic equations and inverted generalized material equations, which express curvature and centerline extension in terms of the internal forces. The resulting set of three first-order differential equations is then numerically integrated using a specific version of the shooting method [2]. In this method, two-point boundary value problems are converted into initial value problems, which are more convenient for solving numerically by finite differences, marching from one boundary point to another.

The proposed beam element has been implemented into an open-source finite element code to ease computations involving more complex structures. The efficiency and accuracy of the developed scheme are documented by numerous examples ranging from circular and parabolic arches to a spiral-shaped beam. The main advantage is that  $h$ -refinement of finite elements, which increases the number of global unknowns and is thus computationally demanding, can be substituted by employing a proper number of segments in the integration scheme, keeping the number of elements and thus also the number of global degrees of freedom limited.

## REFERENCES

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