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WEIGHTED QUADRATURE RULES FOR HIERARCHICAL B-SPLINES

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Weighted quadrature (WQ) has been introduced, in [1], to reduce the number of quadrature points when computing Galerkin integrals for B-spline basis functions. In combination to the sum-factorization and other implementational techniques, it reduces significantly the cost of formation of isogeometric matrices. Further efficiency is possible when only the matrix-vector multiplication is needed, that can be achieved in $O(N\ p^{d+1})$ FLOPS without computing the matrix itself, see [2].

The advantage of WQ is that the number of exactness conditions to be imposed is less than for Gauss quadrature, even for generalized Gauss quadrature as in [3, 4, 5, 6] or reduced quadrature [7, 8]. Indeed, for WQ the number of quadrature points is mildly dependents on the spline degree.

In this work we extend WQ to hierarchical splines. Since hierarchical splines are a selection of standard B-splines on different levels, we can define hierarchical WQ as a linear combination of standard WQ on different tensor-product levels. We present an algorithm for WQ on hierarchical splines of any dimension and we show its performance for adaptive L2 projection on 2D and 3D geometries.

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