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STABLE NUMERICAL RECONSTRUCTION OF NON-SMOOTH BOUNDARY DATA IN STEADY-STATE ANISOTROPIC HEAT CONDUCTION

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We investigate a situation that usually occurs for numerous real life problems in science and engineering, namely the reconstruction of the missing non-smooth thermal boundary conditions (i.e. temperature and normal heat flux) on an inaccessible portion of the boundary of an anisotropic solid from the knowledge of over-prescribed data (i.e. Cauchy data) on the remaining and accessible boundary in the steady-state case. It is well-known that the solution to this inverse Cauchy problem, in a very weak sense in this case, is unstable, provided that such a solution exists. Consequently, a stabilising method is developed herein based on *a priori* knowledge on the solution to this Cauchy problem and the smoothing nature of the corresponding direct problems employed. More precisely, this inverse problem is approached by transforming it into a control one which reduces to solving an appropriate minimisation problem in a suitable function space. The latter is approached by employing an appropriate variational method which yields a gradient-based iterative algorithm that consists, at each step, of two direct and two corresponding adjoint problems. It is worth mentioning that this algorithm is designed to approximate merely L^2-boundary data, hence the notion of solution and the convergence analysis require special attention. The numerical implementation is realised for two-dimensional non-homogeneous anisotropic solids using the finite element method (FEM), whilst regularisation is achieved by ceasing the iterative process according to three stopping regularising criteria.

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