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Performance of Refined Isogeometric Analysis in Solving Generalized and Quadratic Eigenvalue Problems

Ali Hashemian[1*], Daniel Garcia[2], David Pardo[3,1,4] and Victor M. Calo[5]

[1] Basque Center for Applied Mathematics, Bilbao, Spain (ahashemian@bcamath.org)
[2] Institute of Environmental Assessment and Water Research, Barcelona, Spain
[3] University of the Basque Country UPV/EHU, Leioa, Spain
[4] Ikerbasque – Basque Foundation for Sciences, Bilbao, Spain
[5] Faculty of Science and Engineering, Curtin University, Perth, Australia

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We study the performance of refined isogeometric analysis (rIGA) [1] when solving two common types of eigenvalue problems: generalized Hermitian eigenproblems (GHEPs) and quadratic eigenproblems (QEPs). For large problem sizes, the eigencomputation cost is governed by the cost of LU factorization [2], followed by the costs of matrix–vector and vector–vector multiplications in the sense of Krylov projection. rIGA, while conserving desirable properties of maximum-continuity isogeometric analysis (IGA), reduces the interconnection between degrees of freedom by adding low-continuity basis functions. Thus, rIGA provides matrix LU factorizations $\mathcal{O}(p^2)$ faster than IGA, being p the polynomial degree of basis functions. It results in an asymptotic improvement of $\mathcal{O}(p^2)$ in the total eigencomputation cost when using rIGA discretization for sufficiently large problem sizes. In practice, when solving GHEPs, characterized by the linear eigensystem $(\mathbf{A} - \lambda\mathbf{B})\mathbf{u} = \mathbf{0}$, Krylov shift-and-invert eigensolvers exploit the maximum improvement of rIGA discretization thanks to the spectrum slicing technique applied to the real-valued spectrum. On the other hand, QEPs, described as $(\mathbf{K} + \lambda\mathbf{C} + \lambda^2\mathbf{M})\mathbf{u} = \mathbf{0}$, commonly have complex eigenvalues. Thus, the improvement rate of rIGA approach deteriorates in moderate-size problems when increasing the number of requested eigenvalues because of multiple matrix–vector and vector–vector operations. Our numerical tests show rIGA improves with respect to IGA the total eigencomputation cost of QEPs by $\mathcal{O}(p)$ for moderate-size problems when we seek to compute a reasonable number of eigenvalues.

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