

Polytopal discontinuous Galerkin approximation of the fully-coupled thermo-poroelastic problem

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Poroelasticity inspects the interaction among fluid flow and elastic deformations within a porous medium. In several applications in the context of environmental sustainability, such as geothermal energy production and CO_2 sequestration, temperature plays a key role in the description of the physical phenomena. Thus, in order to correctly describe these geological processes, the differential problem should also take into account the influence of the temperature, leading to a fully-coupled thermo-poroelastic (TPE) system of equations [1]. We present and analyze a polytopal discontinuous Galerkin (PolyDG) method for the numerical modelling of the quasi-static fully-coupled thermo-poroelastic problem. The geometric flexibility and the arbitrary-order accuracy featured by PolyDG methods ensure a high-level of flexibility and precision that are needed to properly represent the solutions. As a starting point for the design of the numerical scheme, we adopt a four-field weak formulation inspired by [2], where an additional total pressure variable is added in order to ensure robustness in the quasi-incompressible limit and inf-sup stability. The stability analysis is carried out via two different approaches and error estimates are derived for the resulting semi-discrete formulation in a suitable mesh dependent energy norm. Several numerical results are presented: we investigate the robustness with respect to the quasi-incompressible case and low values of the coupling coefficients, considering manufactured solutions. Finally, a model problem inspired by a geothermal energy production set-up is carried out.

REFERENCES

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