

# A UNIFIED ANALYSIS FRAMEWORK OF NAVIER-STOKES CAHN-HILLIARD MODELS WITH DIFFERENT DENSITIES

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Over the last decades, many diffuse-interface Navier-Stokes Cahn-Hilliard (NSCH) models with different have appeared, for example see [1, 3]. All these models claim to describe the same physical phenomena but are distinct from one another. In this talk [2] we bring these models together by laying down a unified framework of NSCH models. The framework is based on the following three principles: (1) there is only one system of balance laws based on continuum mixture theory that describes the physical model, (2) there is only one natural energy-dissipation law that leads to quasi-incompressible NSCH models, (3) variations between the models only appear in the constitutive choices.

Furthermore, we show the mobility to be of degenerate type and reveal that a non-degenerate mobility leads to incompatibility in the single-fluid regime. Moreover, we aim to highlight and rectify inconsistencies of existing volume-averaged velocity based models with respect to mixture theory.

## REFERENCES

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