

## Space-time IGA

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The concept of k-method was proposed as one of the key features of isogeometric analysis, "a new, more efficient, higher-order concept", in the seminal work [1]. The idea of using high-degree and continuity splines (or NURBS, etc.) as a basis for a new high-order method appeared promising from the beginning, and received confirmations from the following developments. The k-method leads to higher accuracy per degree-of-freedom, but, at the same time, the k-method brings significant challenges at the computational level: using standard finite element routines, its computational cost grows with respect to the degree, making degree raising computationally expensive. However, recent ideas allow a computationally efficient k-method.

In this talk we present our experience (based on [2] and more recent work) on the k-method in Galerkin space-time isogeometric discretization of the heat equation. That is, we use smooth splines in space and time. Exploiting the tensor product structure of the basis functions in the parametric domain, we propose a preconditioner that is the sum of Kronecker products of matrices and that can be efficiently applied thanks to an extension of the classical Fast Diagonalization method. The preconditioner is robust w.r.t. the polynomial degree of the spline space and the time required for the application is proportional to the number of degrees-of-freedom, in our numerical tests with serial execution.

## REFERENCES

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