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Shape Signature Subspace: A shapewise-supervised dimension-reduction approach for shape optimisation

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Simulation-driven shape optimisation problems often suffer from the curse of dimensionality: the computational complexity rises exponentially with the dimension of a design space [1]. One approach to cure this issue is to identify and exploit the latent features, which form the basis of a subspace whose dimensionality is lower than the original one. The commonly used dimension reduction approaches, e.g., the truncated Karhunen-Loève decomposition (KLD) [2] (closely related to Principal Component Analysis (PCA) [3]), auto-encoders [4] and their variants, usually generate subspaces that often fail to preserve the intrinsic geometric structure of the shape, thus resulting in many invalid geometries when explored for shape optimisation [5]. Moreover, the basis of the subspace is solely formulated with geometric features and no information of physics, against which design is optimised, is incorporated. There exist supervised techniques, such as the active-subspace method [6], that can be used to embed physics, but they become computationally intensive as they require the evaluation of gradients of the associated complex physical quantities.

In this work we propose a Shape-Signature Subspace (SSS) technique for dimension reduction, which uses higher-level information for the shape in terms of its geometric integral properties. In our case, these integral properties are geometric moments [7] of varying order, whose usage is based on two fundamental insights: (i) the geometric moments of a shape are intrinsic features of the underlying geometry and (ii) they provide a unifying medium between the shape's geometry and physics. To maximise the geometric information retained in the subspace, we evaluate a set of composite moments by disintegrating the shape into several sub-geometries. Afterwards, we use the divergence theorem to evaluate moments of all the sub-geometries up to a certain order. Once moments are evaluated they are used, along with the shape modification function, to form a shape signature vector (SSV), which act as a descriptor to uniquely represent each instance in the design space. Afterwards, we use SSV to construct a symmetric and positive definite covariance matrix, whose eigendecomposition results in a latent feature matrix. The columns of this matrix are orthogonal eigenvectors, which, along with the highest eigenvalues, span the basis of a subspace retaining the highest variance in terms of geometry, its underlying structure and physics.

The feasibility of the proposed method is tested against the challenging problem of optimising a ship hull, parameterised with 27 design parameters, against its wave resistance coefficient (c_w). The results showed that subspace obtained with KLD/PCA results in 44% of reduction in the original design space dimensionality, whereas 70% of dimension reduction is achieved with SSS. The number of invalid geometries produced by the SSS-based subspace is 36% less compare to the subspace formed with KLD/PCA, and optimisation performed in this subspace converges faster to the optimal solution.

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